Homework 2

Problem 1. Given the temperature of the CMB today, $T_{\rm CMB} = 2.7$K, and the baryon density $\Omega_{\rm b} = 0.01$,

a) calculate the ratio between the number density of photons and that of baryons today:

$$\eta_\gamma \equiv \frac{n_\gamma}{n_{\rm b}}. \tag{1}$$

You may neglect neutrons.

b) Consider η_γ right before the epoch of recombination ($z \lesssim 1100$). Does its value differ from the answer to point a)?

c) At very early times ($kT \gg m_{\rm p}c^2$), baryons were fully relativistic. What do you expect the order of magnitude of η_γ to be back then?

Problem 2. Consider a 1D (classical) harmonic oscillator, with energy given by

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2, \tag{2}$$

and equations of motion

$$\dot{x} = \frac{p}{m}; \qquad \dot{p} = -kx. \tag{3}$$

Show that the collisionless Boltzmann equation for a ensemble of oscillating particles is

$$\frac{\partial f}{\partial t} + \frac{p}{m}\frac{\partial f}{\partial x} - kx\frac{\partial f}{\partial p} = 0. \tag{4}$$

Then show that if the distribution function is a function of energy alone, $f = f(E)$, then it is a solution of the collisionless Boltzmann equation.