Homework 6

Problem (6 points). Write a code, in a programming language of your choice, that generates a one-dimensional Gaussian random field with power spectrum $\mathrm{P}(k) \propto k^{-1}$.

Fist generate the field in Fourier space, then apply an inverse Fourier transform to obtain the field in position space.

Things to keep in mind:

- Fourier coefficients are random complex numbers $\Phi_k = |\Phi_k| e^{i\phi_k}$.
- The modulus $|\Phi_k|$ is drawn from a Gaussian distribution with mean zero and variance given by the power spectrum $\mathrm{P}(k)$.
- The phase ϕ_k is completely random.
- The field in position space $\Phi(x)$ is a real quantity. This means that the value of Φ_k automatically determines Φ_{-k} (find this relation).
- The field should average to zero.
- When using an inverse Fast Fourier Transform, make sure to understand what convention is used for the definition of the input vector in Fourier space (i.e. which Φ_k goes where in the input vector).
- It can be more convenient to work with an odd number of points.
- No need to worry about the units of k or P(k) (for the purposes of this homework).

i) Show the code, with in-line comments.

ii) Plot one realization of the random field.

iii) Do the same with $\mathrm{P}(k) =$ constant and $\mathrm{P}(k) \propto k^{-2}$.

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